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Models for jet power in elliptical galaxies: support for rapidly spinning black holes

R. S. Nemmen,[1⋆] R. G. Bower,[2] A. Babul,[3] and T. Storchi-Bergmann[1]
[1] *Instituto de Física, Universidade Federal do Rio Grande do Sul, Campus do Vale, Porto Alegre, RS, Brazil*
[2] *Department of Physics, Institute for Computational Cosmology, Durham University, South Road, Durham DH1 3LE*
[3] *Department of Physics & Astronomy, University of Victoria, Elliott Building, 3800 Finnerty Road, Victoria, BC V8P 1A1, Canada*

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ABSTRACT
Recently, Allen et al. measured a tight correlation between the Bondi accretion rates and jet powers of nearby X-ray luminous elliptical galaxies. We employ two models of jet powering to understand this correlation and derive constraints on the spin and accretion rate of the central black holes. In these models, the magnetic fields threading the accretion flow or the spinning black hole can extract energy electromagnetically from the disk and/or hole and drive the observed jets. The first is the Blandford-Znajek (BZ) model, in which the spin energy of the hole is extracted; the second model is an hybrid version of the Blandford-Payne and Blandford-Znajek processes, in which the outflow is generated in the inner parts of the accretion disk. We assume advection-dominated accretion flows (ADAF) and account for general relativistic effects, in particular the dependence of the radius of the marginally stable orbit $R_{\rm ms}$ on the dimensionless spin j and enhancement of the magnetic field strength by shear driven by the Kerr metric. We calculate the jet efficiencies $\eta_{\rm jet} \equiv P_{\rm jet}/\dot{M}(R_{\rm ms})c^2$ needed to reproduce the correlation and find that the theoretical maximum values of $\eta_{\rm jet}$ for the BZ and hybrid models are approximately 20% and 50% respectively. Our modelling implies that for typical values of the disk viscosity parameter $\alpha \sim 0.01-1$ the tight correlation implies the narrow range of spins $j \approx 0.7-1$ and accretion rates $\dot{M}_{\rm ms} \approx (0.01-1)\dot{M}_{\rm Bondi}$. Our results provide support for the "spin paradigm" scenario and suggest that the central black holes in the cores of clusters of galaxies must be rapidly rotating in order to drive radio jets powerful enough to quench the cooling flows.

Key words: accretion, accretion disks – black hole physics – galaxies: active – galaxies: jets – X-rays: galaxies – MHD

1 INTRODUCTION

The extragalactic jets launched from radio galaxies are some of the most energetic phenomena in the universe, and their nature has been challenging our theoretical understanding of the physical processes involved since they were first observed, more than thirty years ago. It is thought that the radio jets are accelerated and collimated in the inner regions of the active galactic nuclei (AGN), near the central supermassive black hole, by the large-scale magnetic fields anchored in the accretion disk (e.g., Begelman et al. 1984; Ferrari 1998).

Observations made with the *Chandra* and XMM-Newton observatories of the cores of groups and galaxy clusters have revealed the dramatic impact of the radio jets launched from the central AGNs on the surrounding X-ray emitting gas. The jets deposit large amounts of energy in their environments, blowing cavities or bubbles which have been observed in X-ray images (e.g., Allen et al. 2006, hereafter A06; Birzan et al. 2004; Fabian et al. 2006; Taylor et al. 2006; Rafferty et al. 2006). The jet power that inflates these bubbles can balance the radiative losses of the central intracluster medium and has important consequences for galaxy formation and the growth of supermassive black holes (e.g., Churazov et al. 2002; Dalla Vecchia et al. 2004; Sijacki & Springel 2006).

Recently, A06 measured a tight correlation between the Bondi accretion rates and jet powers of nine nearby, X-ray luminous elliptical galaxies using *Chandra* X-ray observations. This correlation implies that a significant fraction of the energy associated with the material entering the accretion radius is converted to the energy carried by the relativis-

⋆ E-mail: rodrigo.nemmen@ufrgs.br

tic jets. As the origin of jets is likely to be related to the spin of the central black hole (e.g., Rees et al. 1982; Blandford 2002), A06 suggested that the tightness of the measured correlation may imply a narrow range of hole spins for the objects in their sample. It is imperative to understand how the above correlation is established and what constraints on the properties of the central black holes can be derived from it. This is the goal of the present paper.

In this paper, we assume that in the nuclei of the observed elliptical galaxies the accretion flow around the black hole is advection-dominated (advection-dominated accretion flows, hereafter ADAF[1], Narayan 2005; Narayan, Mahadevan & Quataert 1998, hereafter N98; Nemmen et al. 2006). ADAFs are expected to be associated with the production of radio jets (e.g., Rees et al. 1982; Meier 2001; Churazov et al. 2005), as jet production is thought to be supressed in standard thin disks (Livio et al. 1999; Meier 2001; Maccarone et al. 2003). We evaluate two physical models of jet production which relate the spin and accretion rate onto the black hole to the observed jet power: the Blandford-Znajek (BZ) model (Blandford & Znajek 1977), in which magnetic fields threading the hole extract its rotational energy and drive the jet; and the model of Meier (2001) (see also Blandford & Payne 1982; Punsly & Coroniti 1990), in which the fields tap energy from the accretion flow, as well as extract energy from the spinning hole. The novelty of the present work is the incorporation of general relativistic effects which affect the accretion flow behavior and were not fully appreciated in previous works, in particular the dependence of the radius of the marginally stable orbit on the black hole spin. We use these models to understand the nature of the correlation between accretion rates and jet powers and to derive constraints on the black hole spins in the sample of elliptical galaxies studied by A06.

This paper is organized as follows. The correlation obtained by A06 is described in §2. The models for the accretion flow and jet power are described in §3. We model the correlation and derive constraints on the black hole spins and accretion rates in §4. Lastly, §5 contains the discussion and concluding remarks.

2 THE EMPIRICAL RELATION BETWEEN ACCRETION RATE AND JET POWER IN X-RAY LUMINOUS ELLIPTICAL GALAXIES

The Bondi accretion rate $\dot{M}_{\rm Bondi}$ is a simple estimate which assumes that the accretion flow has spherical symmetry and negligible angular momentum (Bondi 1952), and can be written as $\dot{M}_{\rm Bondi} = \pi\lambda c_s \rho r_A^2$, where $r_A = 2GM_\bullet/c_s^2$ is the accretion radius, G is the gravitational constant, $M_\bullet$ is the black hole mass, c_s is the sound speed of the gas at r_A, ρ is the density of gas at r_A and λ is a numerical coefficient that depends on the adiabatic index of the gas. This estimate is frequently used in studies of the central X-ray emitting gas in elliptical galaxies (e.g., Di Matteo et al. 2003; Pellegrini 2005).

A06 measured a tight correlation between $\dot{M}_{\rm Bondi}$ and jet power using *Chandra* X-ray observations of nine nearby, X-ray luminous giant elliptical galaxies. The values of $\dot{M}_{\rm Bondi}$ were calculated from the gas temperature and density profiles observed with *Chandra* and the black hole masses. The black hole masses were obtained from the optical velocity dispersion measurements using the correlation between central black hole mass and velocity dispersion of Tremaine et al. (2002). The nuclei of these objects are extremely sub-Eddington, i.e. they have extremely low accretion rates compared to the Eddington rate, $\dot{M}_{\rm Bondi}/\dot{M}_{\rm Edd} \lesssim 10^{-3}$, where $\dot{M}_{\rm Edd} \equiv 22 M_\bullet/(10^9 M_\odot)\ M_\odot\,{\rm yr}^{-1}$. The jet powers were estimated from the energies and timescales required to inflate cavities observed in the surrounding X-ray emitting gas such that $P_{\rm jet} = E/t_{\rm age}$, where E is the energy required to create the observed bubbles and $t_{\rm age}$ is the age of the bubble. A06 estimates are based on the X-ray bubbles are inflated slowly.

The correlation between $\dot{M}_{\rm Bondi}$ and jet power $P_{\rm jet}$ found by A06 is described by a power-law model of the form

$$\log \frac{P_{\rm Bondi}}{10^{43}\ {\rm erg\ s^{-1}}} = A + B \log \frac{P_{\rm jet}}{10^{43}\ {\rm erg\ s^{-1}}}, \qquad (1)$$

where $P_{\rm Bondi}$ is the total accretion power released for an efficiency of 10%, $P_{\rm Bondi} = 0.1\dot{M}_{\rm Bondi}c^2$. Fitting this power-law to the data A06 obtained $A = 0.65 \pm 0.16$ and $B = 0.77 \pm 0.20$. The power-law model is shown in Figure 1 as the solid line together with the *Chandra* X-ray data used by A06[2]. The correlation implies that a significant fraction ($P_{\rm jet}/(\dot{M}_{\rm Bondi}c^2) = 2.2^{+1.0}_{-0.7}$% for $P_{\rm jet} = 10^{43}\ {\rm erg\ s^{-1}}$) of the energy associated with the rest mass of gas entering r_A is channeled into jet power.

3 MODELS OF THE JET POWER

The nature of the power source of the radio jets in AGN has been the subject of much debate. The basic ingredients in all models for the jet power is a large-scale rotating magnetic field, brought in near the central black hole by the accretion disk (e.g., Blandford 2002).

In the scenario first envisaged by Blandford & Znajek (1977) (the BZ model), large-scale magnetic fields threading the event horizon of the black hole can extract electromagnetically part of the huge rotational energies of the hole, which may ultimately accelerate plasma far away from the hole (e.g., Blandford & Znajek 1977; Thorne et al. 1986). Not all models rely on the spin energy of black holes to power the jets. Blandford & Payne (1982) have proposed an alternative model in which the magnetic field threading the accretion disk extracts energy and angular momentum from the rotating plasma. Punsly & Coroniti (1990) and Meier (1999, 2001)) have incorporated the BZ and Blandford-Payne processes into a single hybrid model in which the jets are powered by both the rotational energy as well as the spinning hole. The spin energy of the hole is extracted by the field that threads the frame-dragged accretion disk inside the ergosphere.

[1] Improved models of ADAFs incorporating winds and convection are also referred as radiatively inefficient accretion flows (RIAF). We simply use the original acronym ADAF.

[2] The data plotted includes the systematic uncertainty of 0.46 dex in $\log P_{\rm Bondi}$, implied by the intrinsic dispersion of 0.23 dex in the $\log M_\bullet - \log\sigma$ relation (see A06).

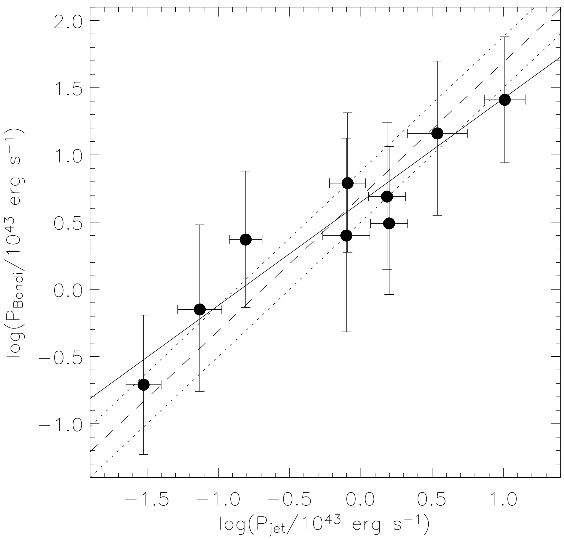

Figure 1. The logarithm of the Bondi accretion power ($P_{\rm Bondi} = 0.1\dot{M}_{\rm Bondi}c^2$) versus the logarithm of the jet power. The fitted power-law model predicted by our jet models ($A = 0.69 \pm 0.19$ and $B = 1$) is represented by the dashed (best-fit) and dotted (error bars) lines, the best-fit power-law model determined by A06 is shown as the solid line.

Numerical general relativistic magnetohydrodynamic (MHD) simulations of accretion flows are beginning to address the possible origin of the jets (e.g., Koide 2003; McKinney & Gammie 2004; De Villiers et al. 2005; Komissarov 2005; Hawley & Krolik 2006). However, given the present uncertainty in our current knowledge regarding the primary energy source for the jets (e.g., Ghosh & Abramowicz 1997; Livio et al. 1999), we consider two models for the jet power in order to assess the correlation found by A06: the BZ model and a hybrid version of the Blandford-Payne and BZ models. In both of these models the spin energy of the hole makes contributions to the jet power, albeit for different reasons.

3.1 Accretion flow structure

We assume that around the black hole is an advection-dominated accretion flow. Several lines of evidence point towards this association between ADAFs and strong radio jets: (1) the magnetic fields associated with ADAFs are much more conductive to the extraction of spin energy from the hole than those associated with standard thin disks (e.g., Rees et al. 1982; Livio et al. 1999; Armitage & Natarajan 1999). (2) X-ray binaries with black hole candidates (XRBs) in the "low/hard" state display a strong correlation between the X-ray emission, which presumably comes from ADAFs, and the presence of radio emission from jets (Gallo et al. 2003; Merloni et al. 2003; Falcke et al. 2004). It is thought that the scaled-up analogs of XRBs in the "low/hard" state are AGNs with low accretion rates, which follow the same correlation (Merloni et al. 2003; Falcke et al. 2004). For higher accretion rates it is observed that the radio-emitting jet is supressed in XRBs in the "high/soft" state and AGNs (Maccarone et al. 2003; Greene et al. 2006). This observed "quenching" of the jet is thought to be caused by a transition in the accretion mode as the accretion rate increases, switching from a ADAF (powerful jets) to a thin disk (weak jets), as invoked in the model of Meier (2001). Furthermore, the observed luminosities of the AGN in elliptical galaxies are typically many orders of magnitude smaller than the luminosity predicted assuming that the Bondi accretion rate is converted to radiation with an efficiency of 10% (e.g., Di Matteo et al. 2003; Taylor et al. 2006). ADAFs in contrast to standard thin disks can explain the observed radiative quiescence of the nuclei of elliptical galaxies and reproduce their nuclear X-ray luminosities with accretion rates comparable to their Bondi rates (Di Matteo et al. 2001, 2003; Loewenstein et al. 2001; Pellegrini 2005; Soria et al. 2006).

We describe the structure of the ADAF using the self-similar equations of Narayan & Yi (1995). We particularly use the analytical equations that describe the vertical half-thickness of the disk H, magnetic field strength B and angular velocity of the disk Ω' (for the corresponding equations, see the Appendix), in terms of the radius R, black hole mass $M_\bullet$, accretion rate onto the black hole $\dot{M}$ and advection parameter f (assumed ≈ 1). We verified that these self-similar solutions are good approximations to the structure of the inner regions of ADAFs even in the presence of winds in the the flow (see §4.3), as in the solution of Blandford & Begelman (1999) (hereafter BB99.

These equations also depend on some properties of the plasma, such as the adiabatic index γ, the ratio of gas to magnetic pressure β and the viscosity parameter α. We calculate γ from β using the expression $\gamma = (5\beta + 8)/3(2 + \beta)$ (Esin et al. 1997). We relate α and β using the equation $\alpha \approx 0.55/(1+\beta)$, obtained from MHD numerical simulations of the evolution of the magnetorotational instability in accretion disks (Hawley, Gammie & Balbus 1995). We assume that the magnetic pressure is related to the field strength as $P_{\rm mag} = B^2/8\pi$.

We introduce three important modifications to the ADAF solutions to take into account general relativistic effects caused by the Kerr metric, not fully considered before in BZ and Blandford-Payne models. With these modifications the accretion flow solutions become strong functions of the black hole spin. Firstly, as the local metric rotates with angular velocity $\omega \equiv -g_{\phi t}/g_{\phi\phi}$ in the Boyer-Lindquist frame (Bardeen et al. 1972, see the Appendix), an outside observer at infinity in the same frame will see the disk and the field rotating with angular velocity $\Omega = \Omega' + \omega$. Secondly, when calculating the field strength at the inner regions of the disk, we take into account the field-enhancing shear caused by the Kerr metric, studied by Meier (1999) and tentatively observed in MHD numerical simulations (Hawley & Krolik 2006). We estimate the azimuthal component of the field as $B_\phi = gB$, g is the field-enhancing factor estimated following Meier (2001) as

$$g \sim 1 + \frac{\omega}{\Omega'}. \qquad (2)$$

The unity term in the above equation ensures that for non-spinning black holes we have $g = 1$ (no field enhancement). Thirdly, the ADAF solutions are evaluated at the marginally stable orbit of the accretion disk, located at the radius $R_{\rm ms}$ which is a function of j (Bardeen et al. 1972, see the Appendix), where $j \equiv J/J_{\rm max}$ ("a/M" in geometrized units, a is the specific angular momentum) is the dimensionless spin parameter of the black hole, J is the angular momentum of the hole, $J_{\max} = GM_\bullet^2/c$ is the maximal angular momentum. We note that Meier (2001) incorporated in his jet power model these same general relativistic effects, but with many simplifications: (a) g is considered as a free parameter, (b) the expression for ω is simplified with respect to ours, (c) the accretion flow solutions are evaluated at two limiting radii: $7GM_\bullet/c^2$ (corresponding to $j \approx 0$) and $1.5GM_\bullet/c^2$ ($j \approx 1$), while we evaluate the solutions at $R_{\rm ms}$ which varies continuously with j.

To relate the poloidal and azimuthal components of the magnetic field, we follow Livio et al. (1999), who argued that the strength of the poloidal field is limited by the vertical extent of turbulent eddies in the disk, such that $B_p \approx H/R\ B_\phi$ where R is the radius in cylindrical coordinates. In the ADAF case this yields $B_p \approx B_\phi$ because $H \sim R$.

3.2 The Blandford-Znajek model

In this model, large-scale magnetic fields thread the horizon of a spinning black hole and are connected to plasma far away located in an "acceleration region" (Blandford & Znajek 1977; Macdonald & Thorne 1982). These fields can extract part of the huge rotational energy of the hole as Poynting flux and/or kinetic energy of the plasma. The available jet power from the BZ process is given by (e.g., Macdonald & Thorne 1982; Thorne et al. 1986)

$$P_{\rm jet}^{\rm BZ} = \frac{1}{32}\omega_F^2 B_\perp^2 R_H^2 j^2 c, \tag{3}$$

where $R_H = (1 + (1-j^2)^{1/2})GM_\bullet/c^2$ is the horizon radius, $B_\perp$ is the strength of the magnetic field normal to the horizon. The factor $\omega_F \equiv \Omega_F(\Omega_H - \Omega_F)/\Omega_H^2$ depends on the angular velocity of the field lines Ω_F relative to that of the hole, Ω_H. We will assume as usual that $\omega_F = 1/2$, which maximizes the power output (e.g., Macdonald & Thorne 1982; Thorne et al. 1986).

Livio et al. (1999) argued that the field threading the horizon is presumably comparable in strength to the field threading the inner regions of the accretion flow. This is also found in numerical simulations of jet formation (e.g., Hirose et al. 2004; McKinney & Gammie 2004). Based on this, we assume that the field strength at the horizon is the same as that at the marginally stable orbit of the accretion disk; therefore, $B_\perp \approx B_p(R_{\rm ms}) \approx g(R_{\rm ms})B(R_{\rm ms})$. As the accretion rate enters the jet power only through the field strength, the jet power depends on the accretion rate measured at the marginally stable orbit of the accretion flow $\dot{M}_{\rm ms} \equiv \dot{M}(R_{\rm ms})$. If the mass is conserved in the disk as in the early version of the ADAF model (Narayan & Yi 1995, N98) then $\dot{M}$ is constant with radius, but recent ADAF models relax this condition (see §3.4).

The jet power resulting from Equation 3 can be written as $P_{\rm jet}^{\rm BZ}(\alpha, j, \dot{M}_{\rm ms}) \propto \dot{M}_{\rm ms}$ and has a complicated dependence on j and α (see the Appendix). An interesting property of the BZ model is that the jet power does not depend on the black hole mass (unless $\dot{M}$ is expressed in units of $\dot{M}_{\rm Edd}$). Figure 2 illustrates the spin dependence of Equation 3, where we define the jet efficiency $\eta_{\rm jet} \equiv P_{\rm jet}/\dot{M}_{\rm ms}c^2$ and adopt $\dot{M}_{\rm ms} = 10^{23}$ g s^{-1} $= 1.6\times10^{-3}\ M_\odot\,{\rm yr}^{-1}$ for the two different values $\alpha = 0.04, 0.3$. As can be seen, the jet power is strongly dependent on the black hole spin. Depending on the value of j, the jet power covers a range of three orders of magnitude. Below we compute the jet power from the hybrid model and show that the dependence of jet power is similar in both models.

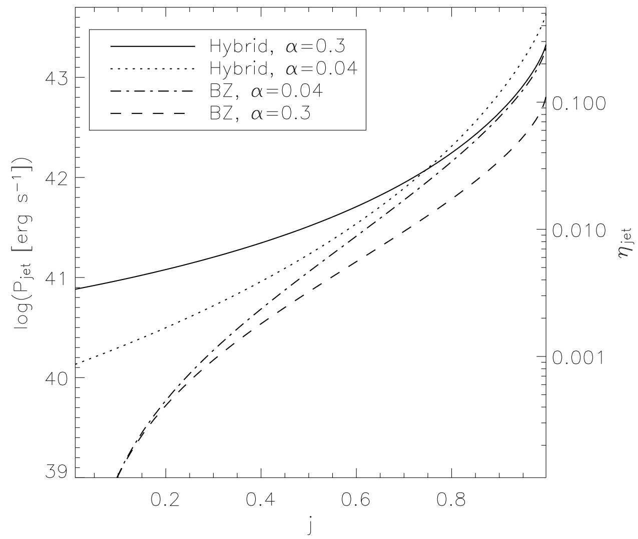

Figure 2. Comparison of the spin dependence of the jet powers predicted by the BZ model (Equation 3) and the hybrid model (Equation 4) for two values of α, taking $\dot{M} = 1.6\times10^{-3}\ M_\odot\,{\rm yr}^{-1}$. The right axis shows the jet efficiency $\eta_{\rm jet} \equiv P_{\rm jet}/\dot{M}c^2$.

3.3 Hybrid model

This model is based on previous work by Meier (2001). It employs a hybrid version of the BZ and Blandford-Payne models, and is similar to the model of Punsly & Coroniti (1990), who studied outflows from the ergosphere. Here the rotating field threads the inner parts of the accretion disk and connects it to regions far away from the hole. The field can extract energy and angular momentum from the accreting matter and can also tap the spin energy of the hole, provided that the field threads a region of the accretion flow extending into the ergosphere. As Meier (2001) discussed, the jet power in this model is a strong function of the thickness of the accretion disk and the black hole spin, such that only when the disk is thick and the hole is rapidly spinning do one obtains strong fields and rapid rotation, and consequently strong jets. This model has been invoked to explain the radio-loud/radio-quiet dichotomy (Meier 2001).

Following Meier (2001) we estimate the electromagnetic output power as

$$P_{\rm jet}^{\rm disk} = (B_\phi H R \Omega)^2 / 32c, \tag{4}$$

where $B_\phi = gB$, $\Omega = \Omega' + \omega$ and the "disk" superscript refers to the fact that the fields thread the accretion disk. All quantities are evaluated at $R = R_{\rm ms}$, assumed to be the approximate characteristic size of the jet-formation region. The resulting jet power evaluated in this way has the form $P_{\rm jet}^{\rm disk}(\alpha, j, \dot{M}_{\rm ms}) \propto \dot{M}_{\rm ms}$. Similarly to Equation 3, $P_{\rm jet}^{\rm disk}$ has a complicated dependence on j and α, and does not depend on the black hole mass. The accretion rate enters the jet power only through the field strength (see the Appendix). Figure 2 shows the spin dependence of Equation 4 for the same parameters used to plot the BZ model. Note that both models give similar results, and that the dependence of $P_{\rm jet}$ on j is much stronger than the dependence on the jet model.

3.4 Basic properties of the models

We note from Figure 2 that the electromagnetic power available from both models is comparable for high values of j, although the BZ power is slightly smaller than the hybrid model. The efficiencies $\eta_{\rm jet}$ predicted by these models are comparable to the Novikov-Thorne thin disk radiative efficiency for high values of j, which is the binding energy of the innermost stable circular orbit (Novikov & Thorne 1973). In particular, the maximal efficiencies (which correspond to $j \approx 0.998$, the limiting value derived by Thorne 1974) in the BZ and hybrid models for $\alpha = 0.04$ are 0.22 and 0.48 respectively; taking $\alpha = 0.3$ the efficiencies are 0.07 (BZ model) and 0.24 (hybrid model). Figure 2 also show that the j-dependence of the jet powers is very steep. This is an important result, which arises from our improvements over previous BZ and Blandford-Payne models, which did not incorporated carefully the physics of the Kerr metric (Ghosh & Abramowicz 1997; Armitage & Natarajan 1999; Meier 2001). A similar j-dependence of the jet power as these ones has been obtained only in complex numerical MHD simulations of jet formation (McKinney 2005; Hawley & Krolik 2006).

Presumably less matter than the amount predicted by the Bondi rate gets down to the black hole for several reasons. As the gas in an ADAF has angular momentum, accretion is driven not just by gravity as in the Bondi flow but also by viscosity, for instance given a certain ambient density in the external medium the accretion rate predicted by the ADAF model is $\dot{M}_{\rm ADAF} \sim \alpha \dot{M}_{\rm Bondi}$ (e.g., N98), with $\alpha < 1$. Furthermore, some part of the gas may be prevented from being accreted due to winds and/or convection (e.g., BB99; Quataert & Gruzinov 2000; Proga & Begelman 2003; Igumenshchev et al. 2003) occuring in the ADAF, reducing even more $\dot{M}_{\rm ms}$ compared to $\dot{M}_{\rm Bondi}$. We allow $\dot{M}_{\rm ms}$ to be smaller than $\dot{M}_{\rm Bondi}$ by introducing the parameter $\epsilon_{\rm Bondi}$, defined as the fraction of $\dot{M}_{\rm Bondi}$ that reaches the black hole such that

$$\dot{M}_{\rm ms} = \epsilon_{\rm Bondi} \dot{M}_{\rm Bondi}. \qquad (5)$$

We emphasize that the $\epsilon_{\rm Bondi}$ parameter measures the fraction of material supplied by the external medium at the Bondi radius that ultimately reaches the innermost stable circular orbit of the accretion flow and gets accreted afterwards. Therefore this parameter encompasses our ignorance about the possible physical processes that may modify the density profile of the accretion flow with respect to the ADAF solution, considering the simplest case where only the density profile is affected. We defer further complications to §4.3.

Noting that $P_{\rm Bondi} \equiv 0.1 \dot{M}_{\rm Bondi} c^2$, each model gives a jet power relation of the form

$$P_{\rm jet}(P_{\rm Bondi}, \alpha, \epsilon_{\rm Bondi}, j) \propto P_{\rm Bondi}. \qquad (6)$$

The main parameters on which the relation $P_{\rm jet}(P_{\rm Bondi})$ depends are thus α, $\epsilon_{\rm Bondi}$ and j. Notice that as mentioned before there is no explicit dependence on the black hole mass and all other quantities (β, γ etc) ultimately depend on these three parameters. Notice that Equation 6 has the same form as the correlation found by A06 (Equation 1) and we can now use the above equation to constrain the parameters of the models using A06's results.

4 CONSTRAINTS ON THE BLACK HOLE SPIN AND ACCRETION RATE

4.1 Results from the values $P_{\rm Bondi}$ vs. $P_{\rm jet}$ measured by Allen et al.

Comparing Equations 1 and 6, it follows that our models predicts the slope $B = 1$, which is somewhat higher than the value obtained by A06, $B = 0.77 \pm 0.20$, although the difference is not statistically significant (see Fig. 1). As A and B in Equation 1 presumably are strongly correlated, we fit to A06's data a power-law model with the fixed value $B = 1$ to find the corresponding value of A. Using the χ^2 fit statistics which accounts only for errors in the values of $P_{\rm Bondi}$ we find $A = 0.69 \pm 0.19$ with $\chi^2 = 2.4$ for 8 degrees of freedom, providing a good description of the data. The power-law model with $A = 0.69$ and $B = 1$ is plotted in Figure 1 as the dashed line; the dotted lines delimitate the corresponding uncertainty in this power-law.

Setting $B = 1$ in Eq. 1 we have

$$A = \log\left(\frac{P_{\rm Bondi}}{P_{\rm jet}}\right) = \log\left(\frac{0.1}{\eta_{\rm jet}\epsilon_{\rm Bondi}}\right), \qquad (7)$$

where $P_{\rm jet}$ is the model jet power. As indicated by Equation 7, the parameter A is a measure of the jet efficiency where $\eta_{\rm jet}$ can be expressed in terms of A as

$$\log(\eta_{\rm jet}) = -(1 + A + \log \epsilon_{\rm Bondi}). \qquad (8)$$

The Equation 7 allows us to obtain the value of A once the values of the parameters α, $\epsilon_{\rm Bondi}$ and j are provided. Using this equation we can derive the range of possible values of $\epsilon_{\rm Bondi}$ and j that reproduce the observed values $A = 0.69 \pm 0.19$ determined above, once α is given.

We constrain the value of α from recent MHD numerical simulations of radiatively inefficient accretion flows, which take into account self-consistently the role of the Maxwell stresses in establishing the value of α. The simulations around Schwarzschild holes of Proga & Begelman (2003) suggest that near the innermost stable circular orbit α reaches high values, $\alpha \approx 0.1 - 0.7$. General relativistic simulations (e.g., McKinney & Gammie 2004; Hirose et al. 2004; Hawley & Krolik 2006) find that in the inner portions of the disk $\beta \approx 1 - 10$. Although α is not explicitly quoted, we can estimate α from these values of β using the relation proposed by Hawley, Gammie & Balbus (1995) (see §3.1), obtaining $\alpha \approx 0.01 - 0.3$. In addition, recent ADAF models of XRBs observations (Quataert & Narayan 1999) also require large values of $\alpha \approx 0.25$.

Figure 3 shows the parameter space (j, $\epsilon_{\rm Bondi}$) that reproduces the observed values of A using the two models of jet power previously discussed, the BZ model (dashed line, §3.2) and the hybrid model (solid line, §3.3). The two panels correspond to two different values of α near the innermost stable circular orbit, $\alpha = 0.3$ ($\beta \sim 1$, left panel) and $\alpha = 0.04$ ($\beta \sim 10$, right panel). The label beside each contour is the value of A for that line ($A = 0.69 \pm 0.19$).

It can be concluded from Figure 3 that the observed

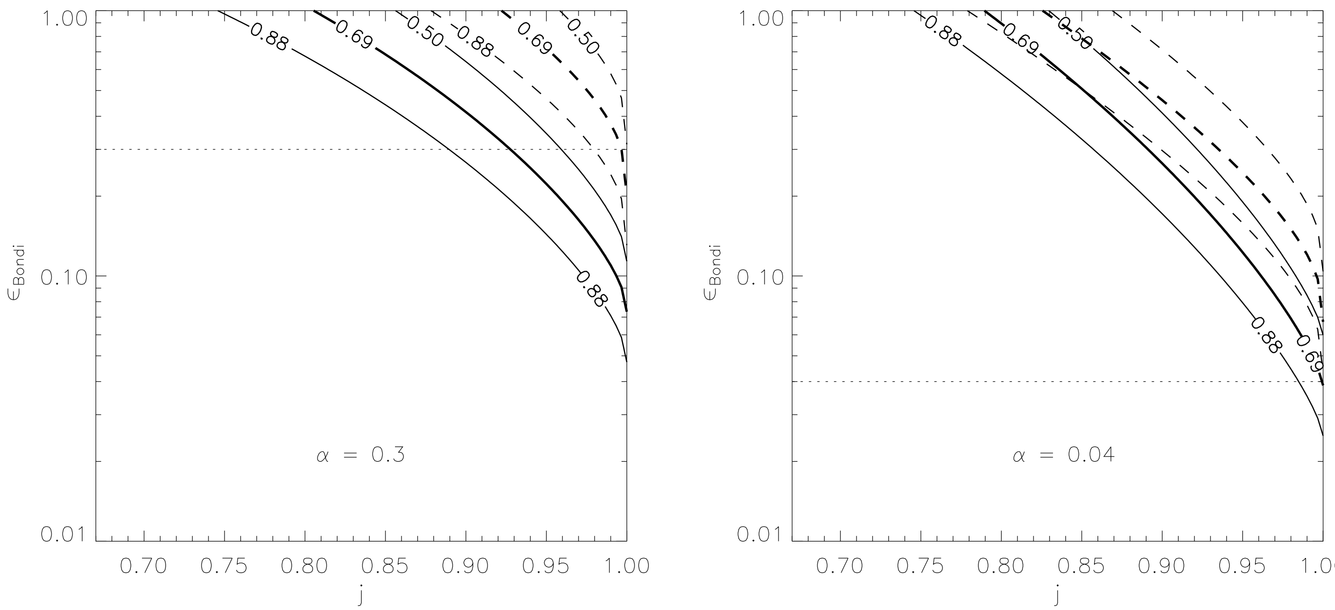

Figure 3. Contours of the parameters $\epsilon_{\rm Bondi}$ and j from the jet models, which reproduce the measured values of A of the empirical correlation between $\dot{M}_{\rm Bondi}$ and $P_{\rm jet}$ (Equation 7). The dashed lines show the predictions of the Blandford-Znajek model and the solid lines show the behavior of the hybrid model. The left panel corresponds to $\alpha = 0.3$ and the right panel to $\alpha = 0.01$. The label beside each contour is the value of A for that line. The thicker lines correspond to the best fitting value of A. The dotted lines indicate the value $\epsilon_{\rm Bondi} = \alpha$ (i.e. $\dot{M}_{\rm ms} = \dot{M}_{\rm ADAF}$).

tight correlation between accretion rates and jet powers implies a narrow range of black hole spins for the elliptical galaxies of the sample of A06, irrespective of the value of α and the model adopted. If $\alpha \approx 0.3$ (Figure 3a) then the hybrid model implies $j \gtrsim 0.75$, while the BZ model implies $j \gtrsim 0.9$; if $\alpha \approx 0.04$ (Figure 3b), both the hybrid and BZ models imply $j \gtrsim 0.75$. These minimal spins are allowed only if all the diffuse hot gas given by the Bondi estimate reaches close to the black hole ($\epsilon_{\rm Bondi} \sim 1$), and they correspond to the minimum jet efficiency $\eta_{\rm jet} \approx 2\%$. Larger spins imply higher efficiencies (see Fig. 2).

More likely, only a fraction of the Bondi rate makes its way to the black hole, because rotating accretion flows predict smaller accretion rates than the Bondi rate for a given value $\rho(r_A)$ (e.g., N98; Proga & Begelman 2003; see §3). It is reasonable to expect that the amount $\dot{M}_{\rm ms} \lesssim \dot{M}_{\rm ADAF}$ reaches the inner part of the disk. The dotted lines in Figure 3 indicate the values $\epsilon_{\rm Bondi} = \alpha$ which correspond to $\dot{M}_{\rm ms} = \dot{M}_{\rm ADAF}$. Considering the case $\alpha \approx 0.3$ (Figure 3a), if $\dot{M}_{\rm ms} \lesssim \dot{M}_{\rm ADAF}$ (or $\epsilon_{\rm Bondi} \lesssim \alpha$) the hybrid model implies $j \gtrsim 0.9$; in the extreme case $j \approx 1$, then $\epsilon_{\rm Bondi} \approx 0.05 - 0.1$ (or $\dot{M}_{\rm ms} \approx (0.2 - 0.4)\dot{M}_{\rm ADAF}$). For the same value of α, the BZ model needs $j \approx 1$ if $\dot{M}_{\rm ms} = \dot{M}_{\rm ADAF}$. In the case where $\alpha \approx 0.04$ (Figure 3b) the best-fit hybrid model is consistent with $\dot{M}_{\rm ms} \approx \dot{M}_{\rm ADAF}$ only for $j \approx 1$ and the BZ model requires $\dot{M}_{\rm ms} > \dot{M}_{\rm ADAF}$. This suggests that if accretion occurs at rates equal or smaller than that specified by the ADAF model, then values of the viscosity parameter $\alpha \lesssim 0.04$ are disfavored. The best-fit BZ model is consistent with $\dot{M}_{\rm ms} \approx \dot{M}_{\rm ADAF}$ only for $\alpha \geqslant 0.2$. We note that the constraints on $\epsilon_{\rm Bondi}$ and j for $\alpha > 0.3$ up to 1 are very similar to those obtained for $\alpha = 0.3$; the exception is if the conditions $\alpha = 1$ and $\dot{M}_{\rm ms} \leqslant \dot{M}_{\rm ADAF}$ are satisfied, in this case then the hybrid and BZ models require $j_{\rm min} \approx 0.75, 0.9$ respectively, these values of $j_{\rm min}$ are slightly smaller than the ones associated with $0.04 \lesssim \alpha \lesssim 0.3$.

The relation 7 was derived based only on the result from the theoretical models of jet powering that $P_{\rm jet} \propto \dot{M}$ (i.e. $B = 1$, §3.2). Using Equation 7 (or Eq. 8) we can plot the possible relation between the jet efficiency $\eta_{\rm jet}$ and $\epsilon_{\rm Bondi}$ as shown in Figure 4. The different lines in Fig. 4 correspond to the different values of A inferred from the observed correlation (see the values beside each line). The bold line is associated with the best-fit value $A = 0.69$. Figure 7 shows that the minimum value of $\eta_{\rm jet}$ needed to account for the observations is $\eta_{\rm jet} = P_{\rm jet}/\dot{M}_{\rm Bondi}c^2 \approx 2\%$, as estimated by A06 under the assumption that $\epsilon_{\rm Bondi} = 1$. This lower value correspond to the smallest allowed spins. The upper limits on the efficiency are associated with maximally spinning black holes ($j \approx 1$), the upper values suggested by the models for $\alpha = 0.04$ are 48% in the case of the hybrid model and 22% in the case of the BZ model (indicated by the dotted line in Fig. 4).

It is possible that accretion may not lead to near-maximal rotation of black holes, as the numerical relativistic MHD simulations of thick-disks of Gammie et al. (2004) have shown. If spin equilibrium is reached at $j \approx 0.93$ as found by Gammie et al. (2004), then an implication is that the black holes in the studied sample of ellipticals must be fed at rates $\epsilon_{\rm Bondi} \gtrsim 0.05$, depending on the value of α and the mechanism of jet powering at work. In particular, if $\alpha \approx 0.3$ the BZ model requires $\epsilon_{\rm Bondi} \approx 0.6 - 1$, with the associated range of allowed spins $j \approx 0.88 - 0.93$ (Figure 3a).

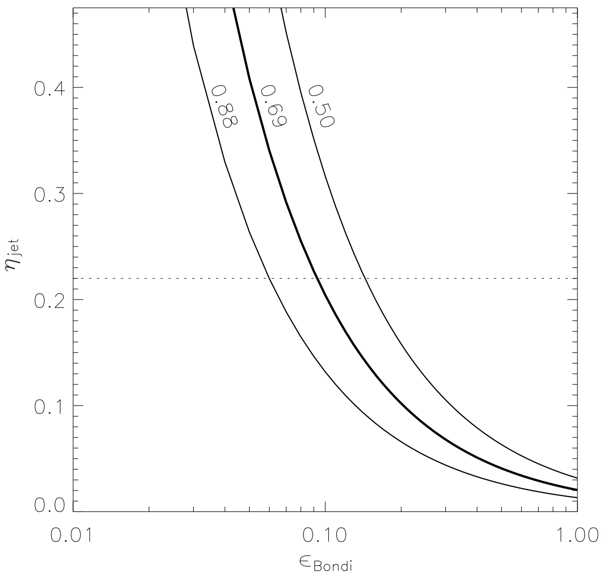

Figure 4. Relation between the parameters $\eta_{\rm jet}$ and $\epsilon_{\rm Bondi}$ required to reproduce the correlation between $\dot{M}_{\rm Bondi}$ and $P_{\rm jet}$. Each line correspond to a different value of A (shown above the lines), the thicker line represents the best-fit value of A. $\eta_{\rm jet}$ ranges up to the theoretical maximum of the hybrid model with $\alpha = 0.04$ (48%). The dotted line corresponds to the maximum efficiency for the Blandford-Znajek model with $\alpha = 0.04$.

4.2 The energy deposited in the cavities

The correlation observed by A06 uses the intracluster medium cavities to estimate the jet power. To derive the energy E required to create the observed bubbles in the X-ray emitting gas, A06 assumes slow expansion rates obtaining the result $E = 4PV$ for $\gamma = 4/3$ (relativistic plasma), where P is the thermal pressure of the surrounding X-ray emitting gas, V is the volume of the cavity and γ is the mean adiabatic index of the fluid within the cavity. A more realistic scenario is one in which the bubbles upon injection expand rapidly in situ to reach pressure equilibrium with their surroundings. In this case, Nusser, Silk & Babul (2006) calculate that the energy E' required to create the cavities is

$$E' \approx 3PV \left(\frac{P_i}{P} \right)^{1/4}, \tag{9}$$

where P_i is the thermal pressure of the surrounding gas at the distance at which the bubble has been injected. Equation 9 suggests that if a bubble is injected with overpressure $P_i/P \gtrsim 10$ then $E' \gtrsim 6PV$. We consider the possibility that as an extreme case the bubble energies are twice the value assumed by A06 ($E' = 8PV$), and calculate the impact of this on the observed jet powers and on the results derived from our models.

We calculate the modified jet powers as $P'_{\rm jet} = E'/t_{\rm age} = 2P_{\rm jet}$ and refit the power-law model with $B = 1$ to the modified data, taking into account the proper error propagation on the values of $P'_{\rm jet}$. We obtain $A = 0.39 \pm 0.19$ with $\chi^2 = 2.4$ for 8 degrees of freedom. As the assumed value of E' implies higher jet powers, our models needs higher values of a to reproduce the increased values of $P'_{\rm jet}$. In particular, if $\alpha \approx 0.3$ these values of A imply $j_{\rm min} \approx 0.84$ for the hybrid model and $j_{\rm min} \approx 0.95$ for the BZ model. Therefore, a narrower range of still large spins is required to explain $P'_{\rm jet}$ and the implied numerical values of $\epsilon_{\rm Bondi}$ and $\epsilon_{\rm ADAF}$ increase slightly.

We should note the possibility that the age of the bubbles calculated by A06 may systematically underestimate the "true" ages. A06 calculates the ages using the formula $t_{c_s} = D/c_s$, where D is the distance of the bubble centre from the black hole and c_s is the adiabatic sound speed, but as discussed by Birzan et al. (2004) (see also Rafferty et al. 2006) there are two other ways of estimating the age of the cavities (Equations 3 and 4 of Birzan et al. 2004) which result in longer timescales when compared to t_{c_s}. To take this into account, we consider the possibility that A06 systematically underestimates the the ages by a factor of 2, such that the modified age is $t_{\rm age} = 2t_{c_s}$ implying the smaller jet power $P''_{\rm jet} = 1/2 P_{\rm jet}$. As a consequence, a systematic decrease in the associated spins occurs; in particular if $\alpha \approx 0.3$ then $j_{\rm min} \approx 0.6$ for the hybrid model and $j_{\rm min} \approx 0.8$ for the BZ model, if $\alpha \approx 0.01$ then $j_{\rm min} \approx 0.6$ for both models. We pointed out before that the energies of the cavities may have been underestimated by up to a factor of 2, this would cancel the effect of the increased ages of the bubbles.

4.3 Potential effects of winds on the jet power

The presence of winds in the accretion flow may cause the removal of mass, angular momentum and energy. As a consequence, ADAFs with winds (ADIOS model, BB99) will have a different dynamical structure than no-wind ADAFs. For instance given the same value of $\dot{M}$ near the black hole, the angular velocity, scale height, total pressure and magnetic field strength predicted by the ADAFs with and without winds will be different. We studied these effects and their impact on jet power as follows.

The ADIOS solution has three parameters in addition to the original ADAF model: p_w, λ_w and ϵ_w. These parameters describe how much mass, energy and angular momentum respectively the wind removes from the accretion flow, and depending on its values we have different types of winds (for more information see BB99). As the jet power in the BZ and the hybrid model depend on the accretion flow solutions H, Ω' and B, we estimate these quantities and the jet power using the ADIOS solution and compare these results with those obtained from the no-wind ADAF model, for appropriate values of the wind parameters. We fixed the parameters common to the two ADAF models in the following values: $\alpha = 0.1$, $\gamma = 1.5$, $M_\bullet = 10^9 M_\odot$ (we verified that different values of these parameters do not change the results below) and varied the wind parameters within the range $p_w = 0-1$, $\lambda_w = 0.1 - 0.75$ and $\epsilon_w = 0.1 - 0.5$. This range of values encompasses all the interesting types of winds.

We obtained that for the adopted range of values of the wind parameters, the solutions H, Ω' and B given by the ADIOS and no-wind ADAF models are only slightly different (the ratio of these solutions are in the range $\approx 0.1 - 3$). Therefore the no-wind solution is a reasonable approximation to the dynamical structure of the ADIOS model. Our calculations show that the ADIOS jet powers are always smaller than the no-wind values regardless of the combination of values of the wind parameters.

As the jet power decreases in the ADIOS solution, the range of allowed black hole spins inferred from the observations of A06 will potentially decrease and the minimal $j_{\rm min}$ will increase. Therefore, the presence of winds in the ADAF will presumably further our conclusion that high values of j are needed in order to reproduce the correlation of A06. The particular values of $j_{\rm min}$ will depend on how much the ADIOS jet power is smaller than the ADAF solution, and this is a function of the wind parameters, whose values are poorly constrained.

5 SUMMARY AND DISCUSSION

We have employed two models to understand the empirical correlation between accretion rates and jet powers of X-ray luminous elliptical galaxies obtained by A06: the Blandford-Znajek model, in which the energy is extracted electromagnetically from the spinning black hole, and a hybrid model which combines the Blandford-Payne and BZ mechanisms, where energy and angular momentum are extracted from the inner parts of the accretion flow, which may suffer frame-dragging. We assume that the accretion flow is advection-dominated (ADAF) and take into account general relativistic effects not fully appreciated before in these models. Our modelling suggests that the dispersion around the best-fit correlation is caused by different values of the black hole spin j and accretion rate of each elliptical galaxy. For typical values of the viscosity parameter $\alpha \sim 0.01-1$, the tight correlation $\dot{M}_{\rm Bondi}$ vs. $P_{\rm jet}$ implies the narrow range of spins $j \approx 0.7-1$ and accretion rates $\dot{M}_{\rm ms} \approx (0.01-1)\dot{M}_{\rm Bondi}$. The inferred range in j and $\dot{M}_{\rm ms}$ is caused by the dispersion in the measured correlation, the uncertainty in the value of α and the adopted model of jet powering. In particular if $\alpha \approx 0.1$ and $\dot{M}_{\rm ms} \lesssim \dot{M}_{\rm ADAF} \sim \alpha \dot{M}_{\rm Bondi}$ then $j \approx 1$, i.e. maximally spinning holes are required.

If winds are present in the ADAF as in the ADIOS model proposed by BB99, and they remove not only mass from the flow but also considerable amounts of energy and angular momentum, then the resulting jet powers are smaller than the ones predicted by no-wind ADAFs. As an outcome hot accretion flows with winds potentially require spins closer to the maximal value compared to no-wind models.

Semi-analytic cosmological simulations of the spin evolution of black holes through mergers and gas accretion (Volonteri et al. 2005) and estimates of the radiative efficiencies of global populations of quasars based on Soltan-type arguments(e.g., Soltan 1982; Yu & Tremaine 2002; Wang et al. 2006) suggest that most nearby massive holes are rapidly rotating. It is reassuring that our results show that rapidly spinning holes with a narrow range of spins are indeed required to account for the powerful jets observed in a handful of elliptical galaxies irrespective of the jet model adopted, in basic agreement with the "spin paradigm" which asserts that radio-loud AGN are associated with rapidly rotating holes and radio-quiet ones with slowly rotating holes.

A06 reported that the efficiency of conversion of $\dot{M}_{\rm Bondi}$ into jet power is significant ($P_{\rm jet}/\dot{M}_{\rm Bondi}c^2 \approx 2\%$ for $P_{\rm jet} = 10^{43}$ erg s^{-1}). In our modelling this corresponds to the case where $\epsilon_{\rm Bondi} = 1$ (i.e. $\dot{M}_{\rm ms} = \dot{M}_{\rm Bondi}$). As presumably $\dot{M}_{\rm ms} \lesssim \dot{M}_{\rm Bondi}$, our models suggest that the efficiency of conversion of the accreting matter into jet power may be even higher. For instance $\eta_{\rm jet}$ may reach up to $\approx 50\%$ in the hybrid model and $\approx 20\%$ in the BZ model for high spins, depending on the adopted model. The extraction of spin energy from the holes is responsible for this noticeable increase in the jet efficiency. We note that the increment of the jet power with black hole spin has been verified in numerical simulations of jets (e.g., De Villiers et al. 2005; McKinney 2005; Hawley & Krolik 2006).

The uppper limit we have obtained for $\eta_{\rm jet}$ using the BZ model is more than ten times higher than the value 0.01 reported by Armitage & Natarajan (1999). This result is mainly due to the fact that we have included a Kerr metric shear-driven dynamo (Meier 1999) which enhances the field strength and was not appreciated by these authors (see §3.1 and Appendix). Based on the calculations of Armitage & Natarajan (1999), Cao & Rawlings (2004) argue that if ADAFs are considered then the BZ mechanism is unable to explain the jet powers of a sample of low accretion rate 3CR FR I radio galaxies observed with the Hubble Space Telescope (jet powers in the range $\sim 10^{41}-10^{45}$ erg s^{-1}). With our new efficiencies, using the critical accretion rate $\dot{M}_{\rm crit} \sim \alpha^2$ (Esin et al. 1997) above which the ADAF solution ceases to be valid, and taking $j = 0.998$ and $\alpha = 0.3$, the BZ model yields $P_{\rm jet} \approx 10^{46}$ erg s^{-1}, which is enough to power the observed jets in the sample used by Cao & Rawlings (2004). It can thus be concluded that ADAFs via the BZ process are able to attain very high jet powers.

The adopted model for the jet-formation mechanism predicts a linear relation between accretion rate and jet power, while the measured best-fit slope of the relation measured by A06 is marginably better fit by a non-linear relationship. We note that if $\epsilon_{\rm Bondi}$ has a weak dependence on $P_{\rm jet}$ the agreement of the model with the observed correlation is improved. We verified that if $\epsilon_{\rm Bondi} \propto P_{\rm jet}^{0.2}$ the slope predicted by the model agrees with the observed best-fit slope. This dependence $\epsilon_{\rm Bondi}(P_{\rm jet})$ might be caused by feedback effects of the jet on the interstellar medium.

The correlation between $P_{\rm Bondi}$ and $P_{\rm jet}$ of A06 and our conclusion that it implies large black hole spins is probably a general result valid for all elliptical galaxies. If the correlation holds in a time-averaged sense for the larger central radio galaxies of galaxy clusters, our results suggest that they have the most powerful jets because they have a ADAF duty cycle long enough, during which the black hole is fed with higher accretion rates and consequently have stronger magnetic fields in the inner regions of the accretion flow, which boost the jet power. Our modelling implies that the spin of all these sources must be close to maximal. ADAFs are required in these sources in order to generate strong poloidal magnetic fields near the hole, needed to achieve the observed jet powers.

Our results reveal a potentially fundamental connection between black holes and the formation of the most massive galaxies: the central holes in the cores of galaxy clusters must be rapidly rotating, in order to make the radio jets powerful enough to provide an effective feedback mechanism and quench the cooling flows, therefore preventing star formation and explaining the observed galaxy luminosity function (e.g., Croton et al. 2006; Bower et al. 2006).

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APPENDIX A: DERIVATION OF THE JET POWER

We list the equations we used to compute the dependence of the jet power on α, j and $\dot{M}$ using the Blandford-Znajek model (§3.2) and the hybrid model (§3.3). The jet power is given by Equations 3 (BZ model) and 4 (hybrid model). The code that implements the equations described in this work and returns the jet power is available at the URL http://www.if.ufrgs.br/~rns/jetpower.htm.

The following equations describe the self-similar ADAF structure (Narayan & Yi 1995), where we use the black hole mass in solar units ($m = M_\bullet/M_\odot$), accretion rates in Eddington units ($\dot{m} = \dot{M}/\dot{M}_{\rm Edd}$, $\dot{M}_{\rm Edd}$ is the Eddington accretion rate defined in §2) and radii in Schwarzschild units ($r = R/(2GM_\bullet/c^2)$):

$$\Omega' = 7.19 \times 10^4 c_2 m^{-1} r^{-3/2} \ \mathrm{s}^{-1}, \tag{A1}$$

$$B = 6.55 \times 10^8 \alpha^{-1/2} (1-\beta)^{1/2} c_1^{-1/2} c_3^{1/4} m^{-1/2} \dot{m}^{1/2} r^{-5/4} \ \mathrm{G}, \tag{A2}$$

$$H/R \approx (2.5 c_3)^{1/2}. \tag{A3}$$

The constants c_1, c_2 and c_3 are defined as

$$\begin{aligned} c_1 &= \frac{5+2\epsilon'}{3\alpha^2} g'(\alpha,\epsilon'), \\ c_2 &= \left[\frac{2\epsilon'(5+2\epsilon')}{9\alpha^2} g'(\alpha,\epsilon') \right]^{1/2}, \\ c_3 &= c_2^2/\epsilon', \\ \epsilon' &\equiv \frac{1}{f}\left(\frac{5/3-\gamma}{\gamma-1} \right), \\ g'(\alpha,\epsilon') &\equiv \left[1 + \frac{18\alpha^2}{(5+2\epsilon')^2} \right]^{1/2}. \end{aligned}$$

The relations among α, γ and β are given in §3.1. The angular velocity of the field seen by an outside observer at infinity in the Boyer-Lindquist frame is $\Omega = \Omega' + \omega$, where the angular velocity of the local metric is given by (Bardeen et al. 1972)

$$\omega \equiv -\frac{g_{\phi t}}{g_{\phi\phi}} = \frac{2aM_\bullet}{a^2(R+2M_\bullet)+R^3}, \tag{A4}$$

using geometrized units ($G = c = 1$).

We estimate the field-enhancing shear caused by the Kerr metric following Meier (2001) as $g \sim 1 + \omega/\Omega'$, such that the azimuthal component of the field is given by $B_\phi = gB$ (see §3.1). The poloidal component is related to the azimuthal component following Livio et al. (1999) as $B_p \approx H/R \, B_\phi \approx B_\phi$.

Lastly, we insert all the quantities defined above into Equations 3 and 4, and then evaluate the resulting equations at the marginally stable orbit of the accretion disk $R_{\rm ms}$, given by (Bardeen et al. 1972)

$$R_{\rm ms} = M_\bullet \left\{ 3 + Z_2 - [(3-Z_1)(3+Z_1+2Z_2)]^{1/2} \right\}, \tag{A5}$$

$$Z_1 \equiv 1 + (1-j^2)^{1/3} \left[(1+j)^{1/3} + (1-j)^{1/3} \right],$$

$$Z_2 \equiv (3j^2 + Z_1^2)^{1/2}.$$

Taking $R = R_{\rm ms}$ in the BZ model corresponds to assume that the strength of the magnetic field at the horizon of the hole is very similar to the corresponding strength at the marginally stable orbit. This is a reasonable assumption according to recent numerical simulations of jet formation (e.g., Hirose et al. 2004; McKinney & Gammie 2004).